Liquid Film Drain from an Accelerating Tank Wall

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Nomenclature

g = acceleration body force
t = time
u = the x component of velocity
x = distance measured along the tank wall from the position of the bulk liquid interface at $t = 0$
y = distance measured perpendicular to the tank wall
δ = thickness of draining film
μ = dynamic viscosity of liquid
ρ = density of liquid
ρ_v = density of vapor

In many studies of a draining cryogenic tank, such as those of wall heat transfer or tank pressurization, it is important to have a knowledge of the liquid film that clings to the tank wall. Such a film is illustrated in Fig. 1. Several investigators have examined a draining film for various reasons in the past. This work has been both analytical and experimental and is described by van Rossum.[1] It is all for constant g.

Neglecting inertia terms in the momentum equation, the velocity in such a laminar draining film with no slip at the wall and zero shear at the liquid-vapor interface is given by[2]

$$u = \frac{(\rho - \rho_v)g}{\mu}\left(y\delta - \frac{y^2}{2}\right) \tag{1}$$

The continuity equation written for a slab of incompressible liquid dx in thickness is

$$\frac{\partial}{\partial x}\int_0^\delta u\,dy + \frac{\partial \delta}{\partial t} = 0 \tag{2}$$

One should note in passing that mass transfer from the liquid film introduces a nonzero term into the right-hand side of Eq. (2).

Substitution of the velocity distribution from Eq. (1) into the continuity equation yields

$$\frac{(\rho - \rho_v)g}{\mu}\,\delta^2\frac{\partial \delta}{\partial x} + \frac{\partial \delta}{\partial t} = 0 \tag{3}$$

Equation (3) is a quasi-linear, partial differential equation. The solution of such equations using the method of characteristics is described by Hildebrand.[3] In this case, application of the method reduces to the solution of the following two ordinary differential equations:

$$\frac{dx}{(\rho - \rho_v)g\delta^2/\mu} = \frac{dt}{1} = \frac{d\delta}{0} \tag{4}$$

The third term of this equation signifies that δ is a constant along a characteristic. This fact simplifies the integration of the equation formed by the first and second terms. This equation is

$$dx = \frac{(\rho - \rho_v)g\delta^2}{\mu}\,dt \tag{5}$$

Integration with the boundary condition $x = 0$ at $t = 0$ yields the desired film thickness

$$\delta = \left(\frac{\mu x}{(\rho - \rho_v)\int_0^t g\,dt}\right)^{1/2} \tag{6}$$

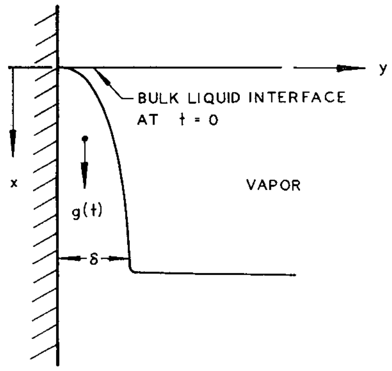

Fig. 1 Physical model.

For a constant g, Eq. (6) reduces to

$$\delta = \left(\frac{\mu x}{(\rho - \rho_v)g\,t}\right)^{1/2} \tag{7}$$

This agrees with the result of van Rossum except that he has neglected ρ_v in comparison to ρ.

Equation (6) is an interesting result since it reveals that the factor of importance in the film profile is the area under the g vs time curve. According to this analysis, which neglects surface tension and contact angle, the profile is unchanged during periods of zero g.

References

[1] van Rossum, J. J., "Viscous lifting and the drainage of liquids," Appl. Sci. Res. **A7**, 121–144 (1958).
[2] Sparrow, E. M. and Siegel, R., "Transient film condensation," J. Appl. Mech. **81**, 120–121 (1959).
[3] Hildebrand, F. B., *Advanced Calculus for Applications* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963), Chap. 8, pp. 379–389.

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Calculation of Natural Modes of Vibration for Free-Free Structures in Three-Dimensional Space

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Having had occasion to require the calculation of natural free-free modes for large three-dimensional structures, we have devised a procedure for developing the free-free matrix for a structure in three dimensions employing lumped mass and inertia concepts. The extension of this concept to three dimensions was indicated in Ref. 1.

Consider a three-dimensional structure consisting of lumped masses and inertias clamped at some point (the origin). Each of the lumped mass-inertia elements that constitute the structure possesses six degrees of freedom; three translational and three rotational. A matrix of flexibility influence coefficients $[C]$ exists which relates the six deflections and rotations of each lumped mass-inertia to forces and moments applied at every lumped mass of the structure. A right-hand coordinate system is employed.

The eigenvalue problem to be solved for the cantilevered modes of the structure can be written as

$$\{\delta\} = \omega^2[C]\lceil M \rfloor\{\delta\} \tag{1}$$

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